HW 1

Exercise 1. Let $A = (1,2)$, $B = (3,1)$. Draw the points $A+B$, $A+2B$, $A+3B$, $A-B$, $A-2B$, and $A-3B$ on a sheet of graph paper (or a reasonably drawn set of axes).

Exercise 2. Which of the following pairs of vectors are perpendicular?

- $(1,-1,1),(2,1,5)$
- $(1,-1,1),(2,3,1)$
- $(-5,2,7),(3,-1,2)$
- $(\pi,2,1),(2,-\pi,0)$

Exercise 3. Suppose $A = (a_1, a_2, a_3)$ is perpendicular to every vector X. Show that A is the zero vector. (Hint: if this holds for every X, it holds in particular for E_1, E_2, and E_3)

Exercise 4. Determine the interior angles of the triangle whose vertices are $(2,-1,1)$, $(1,-3,-5)$, and $(3,-4,-4)$. (Hint: label the points as P, Q, and R. Then, for instance, one of the angles can be found by computing the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Then you can do this for the other angles.)

Exercise 5. Let $A_1, \ldots, A_r$ be *nonzero* vectors which are mutually perpendicular (i.e. $A_i \cdot A_j = 0$ whenever $i \neq j$). Suppose $c_1, \ldots, c_r$ are numbers such that

$$c_1 A_1 + \cdots + c_r A_r = 0.$$

Show that we must have $c_i = 0$ for each $i = 1, \ldots, r$.

Exercise 6. Let $P = (1,3,-1)$ and $Q = (-4,5,2)$. Determine the coordinates of the following points

- The midpoint of the line segment between P and Q
- The point on this line segment that is two thirds of the way from P to Q.

Exercise 7. Find the equation of the plane passing through the points $(2,1,1)$, $(3,-1,1)$, and $(4,1,-1)$. (Hint: to obtain a normal vector to this plane, label the points P, Q, and R and form the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. What is true of $\overrightarrow{PQ} \times \overrightarrow{PR}$?)

Exercise 8. Find a parametric representation for the line of intersection of the planes

$$\begin{aligned} 2x + y + 5z &= 2 \\ 3x - 2y + z &= 3. \end{aligned}$$

(Hint: notice that when two planes intersect, the line of intersection is perpendicular to the normal vectors of both planes. It might help to sketch a picture.)

Exercise 9. Compute the area of the parallelogram spanned by the vectors $(3,-2,4)$ and $(5,1,1)$.