HW 3

Exercise 1. Sketch the level curves for the following functions (just pick a few values for f).

- $f(x, y) = xy$.
- $f(x, y) = (x-1)(y-2)$ (think of the relation this has with the previous function).
- $f(x, y) = 2x - 3y$.

Exercise 2. Find the partial derivatives $\partial f/\partial x$, $\partial f/\partial y$, and $\partial f/\partial z$ (where it applies) for the following functions.

- $f(x, y, z) = e^{xyz}$.
- $f(x, y, z) = \sin(xy) + \cos(z)$.

Exercise 3. Find the gradient of $f(x, y, z) = e^{3x+y}\sin(5z)$ at $(0, 0, \pi/6)$.

Exercise 4. Let $A = (a_1, a_2, a_3)$ and let f be the function on $\mathbb{R}^3$ defined by $f(X) = A \cdot X$ (since we're in $\mathbb{R}^3$, one can write $X = (x, y, z)$). What is grad f?

Exercise 5. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined on an open set U. Let $X \in U$. Suppose that we have a vector A and a function $g(H)$ defined for all H in a neighborhood of O (i.e. for small H) such that

$$f(X + H) - f(X) = A \cdot H + \|H\| g(H).$$

Show that $A = \operatorname{grad} f$. (Hint: use special values of H, such hE_i, where $h \in \mathbb{R}$ is small and E_i is the i-th standard basis vector.) (Note: the point of this exercise is to show that you actually don't need to assume that the partial derivatives exist in the definition of differentiability; the above condition already ensures they exist, and that they in fact form the components of this vector A.)

Exercise 6. Let $f(x, y) = e^{9x+2y}$ and $g(x, y) = \sin(4x + y)$. Suppose $C(t)$ is a differentiable curve with $C(0) = (0, 0)$. Given:

$$\frac{d}{dt} f(C(t))\bigg|_{t=0} = 2, \ \frac{d}{dt} g(C(t))\bigg|_{t=0} = 1.$$

Find $C'(0)$. (This problem is essentially saying the following: if f and g are appropriately chosen functions, and I tell you how the values of f and g are changing as $C(t)$ goes through a point at time t, then you work out the direction $C(t)$ is headed at time t.)