**HW 6.**

**Exercise 1.** Let $F(X) = r^n X$, where $r = \|X\|$, the distance to the origin. Find a potential function for $F$. (Hint: recall that for a function $f = g(r)$, the gradient is

$$\operatorname{grad} f(X) = \frac{g'(r)}{r} X.)$$

**Exercise 2.** Let $r = \sqrt{x^2 + y^2}$. Compute the integral of

$$F = \frac{1}{r} X$$

around the circle of radius 2 centered at the origin, going counterclockwise.

**Exercise 3.** Let

$$G(x, y) = \left( \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right).$$

Compute the integral of $G$ along the following curves:

- $x^2 + y^2 = 2$ from $(1, 1)$ to $(-\sqrt{2}, 0)$.
- Counterclockwise around the entire circle $x^2 + y^2 = r^2$ where $r$ is some fixed radius.

**Exercise 4.** Integrate the field $F = (x^2 y^2, xy^2)$ counterclockwise around the closed path formed by the parts of the line $x = 1$ and the parabola $x = y^2$.

**Exercise 5.** Let $r = \sqrt{x^2 + y^2}$ and

$$F(x, y) = \left( \frac{x}{r^3}, \frac{y}{r^3} \right).$$

Find the integral of $F$ along $C(t) = (e^t \cos t, e^t \sin t)$ from $(1, 0)$ to $(e^{2\pi}, 0)$. (Hint: do you really want to evaluate this directly?)

**Exercise 6.** Find potentials for the following vector fields.

- $(y \sin(z), x \sin(z), xy \cos(z))$
- $(z^2, 2y, 2xz)$