HW 7.

Exercise 1. Evaluate the following repeated integrals.

- $$\int_0^{\pi} \int_0^{\sin(x)} y \; dydx$$
- $$\int_0^1 \int_{y^2}^{y} \sqrt{x} \; dxdy$$

Exercise 2. Sketch the regions

- $$R_1 = \{(x, y) \in \mathbb{R}^2 : |x| \leq 1, -1 \leq y \leq 2\}$$
- $$R_2 = \{(x, y) \in \mathbb{R}^2 : x + y \leq 1, x \geq 0, y \geq 0\}$$
- $$R_3 = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$$

Note that

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0, \end{cases}$$

so for the last one, it may help to consider what happens when $x \geq 0, y \geq 0$, then what happens when $x \geq 0$, $y \leq 0$, etc.

Exercise 3. Compute the integral

$$\iint_R x \cos(x + y) dA$$

where R is the inside of the triangle whose vertices are $(0, 0)$, $(\pi, 0)$, (π, π).

Exercise 4. Compute the integral

$$\iint_R (x^2 - y^2) dA$$

where R is the region defined by $0 \leq x \leq 1$, $x^2 - y^2 \geq 0$.

Exercise 5. Integrate $e^{-(x^2+y^2)}$ over the disk bounded by the circle $x^2 + y^2 = a^2$, where $a > 0$.

Exercise 6. Let $a > 0$. Evaluate

$$\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dxdy$$

by changing to polar coordinates.

Exercise 7. A cylindrical hole of radius 1 is bored through the center of a sphere of radius 2. What volume is removed?