HW2 All of the following exercises are from LADW.

2.2. Find *all* solutions of the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0},$$

where $\mathbf{v}_1 = (1,1,0)^T$, $\mathbf{v}_2 = (0,1,1)^T$ and $\mathbf{v}_3 = (1,0,1)^T$. What conclusion can you make about linear independence (dependence) of the system of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?

2.1. Write the systems of equations below in matrix form and as vector equations. Find the solutions in vector form.

(a)

$$\begin{cases} x_1 - 2x_2 - x_3 = -1 \\ 2x_1 + 2x_2 + x_3 = 1 \\ 3x_1 - 5x_2 - 2x_3 = -1 \end{cases}$$

(b)

$$\begin{cases} x_1 + 2x_2 + 2x_4 = 6 \\ 3x_1 + 5x_2 - x_3 + 6x_4 = 17 \\ 2x_1 + 4x_2 + x_3 + 2x_4 = 12 \\ 2x_1 - 7x_3 + 11x_4 = 7 \end{cases}$$

3.1. For what value of b does the system

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 4 \\ b \end{pmatrix}$$

have a solution? Find the general solution of the system for this value of b.

3.2. Determine whether the following vectors are linearly independent:

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

3.4 Do the polynomials x^3+2x, x^2+x+1, and x^3+5 span $\mathbb{P}_3$? Justify your answer. (Hint: $\mathbb{P}_3$ is isomorphic to $\mathbb{R}^4$ via the linear map $A : \mathbb{R}^4 \to \mathbb{P}_3$ given by $A(a_0, a_1, a_2, a_3)^T = a_0+a_1x+a_2x^2+a_3x^3$.)

3.6 Prove or disprove: If a square matrix A has linearly independent columns, then so does $A^2 = AA$.

3.8 Show that if $A\mathbf{x} = \mathbf{0}$ has a unique solution (i.e. only the trivial solution), then A is left-invertible. Note that we've already seen that if A is left-invertible, then there is a unique solution to $A\mathbf{x} = \mathbf{0}$; now we are going the other way around. (Hint: elementary matrices/row operations are invertible, so you can reduce the problem to showing that a matrix A with columns that are *distinct* standard basis vectors is left-invertible.)